

The Use of MathTrax in Algebra Teaching

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Abstract

Interest in using technology in teaching has seen tremendous growth in the last two decades. However, the popular device that has been broadly used in mathematics classrooms is the graphing calculator in which numerical solutions and graphs are displayed for visualizing but neither for meanings or making connections nor for interactions. This study carried out an experiment that promoted interactive and collaborative learning and provided students opportunities to make sense of graphing functions with the use of computer graphing software – MathTrax. Although similar in concept with the graphing calculator, MathTrax with its unique audio and visual feature has shown rich in ideas and applications. The study illustrates the eighth grade students' ability to learn the new software with peers as well as individually and delineates the benefits of the use of this free-download computer graphical software on motivating learning and enhancing students' concept of algebraic functions.

1. Introduction

With the support of the National Council of Teachers of Mathematics (NCTM), the use of technology has become popular in the mathematics classroom. NCTM believes “technology is an essential tool for learning mathematics in the 21st century and all schools must ensure that all their students have access to technology” (NCTM website, 2008). As so, teachers at middle and high school grade levels are making use of different types of technology to help move their students to the student-centered style of learning. Students can follow Piaget’s inquiry-based style of learning to help them develop their own understanding of mathematical concepts. The national and state standards and the various research studies support the use of interactive technology to lead students to advance their problem solving skills and increase their math comprehension. Indeed, research results have delineated that students’ levels of understanding and mathematics achievement elevate along with the effective use of technology in the classroom. Of that interest, the author of this article approached an interactive learning environment in which audio and visual graphical software – MathTrax – was used to enhance students’ graphing skills and motivate their collaborative work, thinking and learning attitudes over the concept of functions. Since the unique features of the software itself would help weaving the purpose of the study, it is worthwhile to elaborate the software at this point.

2. What is MathTrax and where was it originated from?

MathTrax is a software application program similar to a graphing calculator, but with more features geared toward helping students, who are visually impaired, learn and grasp complex mathematical concepts and the graphs of common functions. MathTrax program is sponsored by NASA Learning Technologies (NLT) (see NASA MathTrax website). The software itself was designed by Robert Shelton who was born with congenital glaucoma and went blind when he was eleven (Shelton, 2009). He now works at the Johnson Space Center leading the Learning Technologies Project and helping to develop software which is designed to aid in the instruction of math, science, and robotics to students with visual impairments. He has his doctorate in mathematics from Rice University and has also worked as a professor. While he wanted to help increase the success of students similar to him in math and science, he also hopes to encourage more of them to pursue careers in math and science. His goal was to make graphing accessible and affordable to people, even to those cannot see. With the MathTrax, not only has he done that for visually impaired learners, but he has also made graphing more accessible to people who are auditory learners and who learn best through multiple modalities.

MathTrax as an audio and visual graphical tool: MathTrax graphs functions similar to a graphing calculator. However, it also has the sound/audio feature which helps learners distinguish between the different parts of the graph and hear what the graph shape is like. The audio description of the graph is an additional feature that makes the software unique in comparison with other graphing calculators or programs. The audio can be played so that someone with visual impairments can hear the description as well as the sound associated with the graph. The verbal description includes key points on the graph or key aspects which students should know including maximum and minimum points, slope, angle of inclination, x and y intercept, and any vertical, horizontal, and slant asymptotes. The sound of the graph uses different tones and pitches to distinguish between increasing and decreasing y values as one moves across the graph from left to right. A *static* noise indicates negative y values, a *bell* indicates the crossing of the y , and a *chirp* indicates where the line or curve crosses the x axis. MathTrax also has a feature allowing the user to zoom in or zoom out on the graph, to re-center the graph about a different point, and to print the graph. All of these features are easily used with buttons at the bottom of the screen, making the program fully functional and user friendly.

In addition to the graphing features of MathTrax, there are two other advantageous aspects which include a data tab and physics tab. Under the data tab, one can upload a data file, and MathTrax will graph it on the coordinate plane. MathTrax also creates a line graph from those data points. This feature enables the program to analyze changes over time or look at trends in data values. Under the physics tab, one can simulate a roller coaster and a rocket launch. In simulating a roller coaster, the function of the track, friction, gravity, and so on can be set to the specific values as desired. This roller coaster car will run the track testing each newly built ride. Students can challenge each other to make it to the end of a given track, or they can create a roller coaster within the program modeled of one in life, simulate the friction and gravity, and finally make adjustments to see how to build a similar roller coaster on the moon where there is less gravity. With the rocket launch, different aspects of a launch are graphed and can be shown simultaneously so that students can compare the relationship they have to each other. Some of the other aspects that MathTrax graphs included are acceleration, altitude, drag, and the rate or speed. Factors such as the weight, material, length of the launch rail, amount of fuel, and angle

of the launch can also be adjusted in order to get the most efficient launch possible. As one can see, there are many features of MathTrax for students to explore and learn from. The advantage to all of these is the ability to play an audio version of the shape of the graph to help aid in the understanding of the meaning of the graphs.

3. Purpose of the study

The power of MathTrax graphing program along with its unique audio feature which is free to all users intrigues teachers, who desire to improve students' graphing skills and functional concepts, to adopt it for classroom using. Of that interest, this study was designed to aim at discovering how this audio and visual enabled graphing software enhance student's learning attitude. While implementing this study, the researcher prospect that if eighth grade students can interact with the software and are truly engaged in the collaborative process with peers, it has the potential to help improve students' independent learning and explorations of the functions taught within algebra courses. The study mainly looked at student reactions to the software, and how much they initially bought into it and what value they placed on it as a peer group. The study also evaluates student's willingness to learn new software to aid in the understanding of graphing functions.

Throughout this research, the researcher desired to explore whether the MathTrax software should be using throughout the school year in teaching algebra particularly. The researcher also seeks to gain insight into how quickly students pick up the program and what tricky points that teachers who adopt the software for classroom teaching need to be aware of, or conscious of, in order to prevent students from making common mistakes. In preparing the activities and designing this study, the researcher anticipated that students would quickly pick up on how to use the software as the interface is simple, user friendly, and well laid out for beginners. The most challenging aspect of learning the software was the format required for a data file and in how to effectively make use of the science applications – rocket launch and roller coaster simulations under the physics tab. The following are a few reviews of research that indicate the benefit of the use of interactive technology in math classroom teaching.

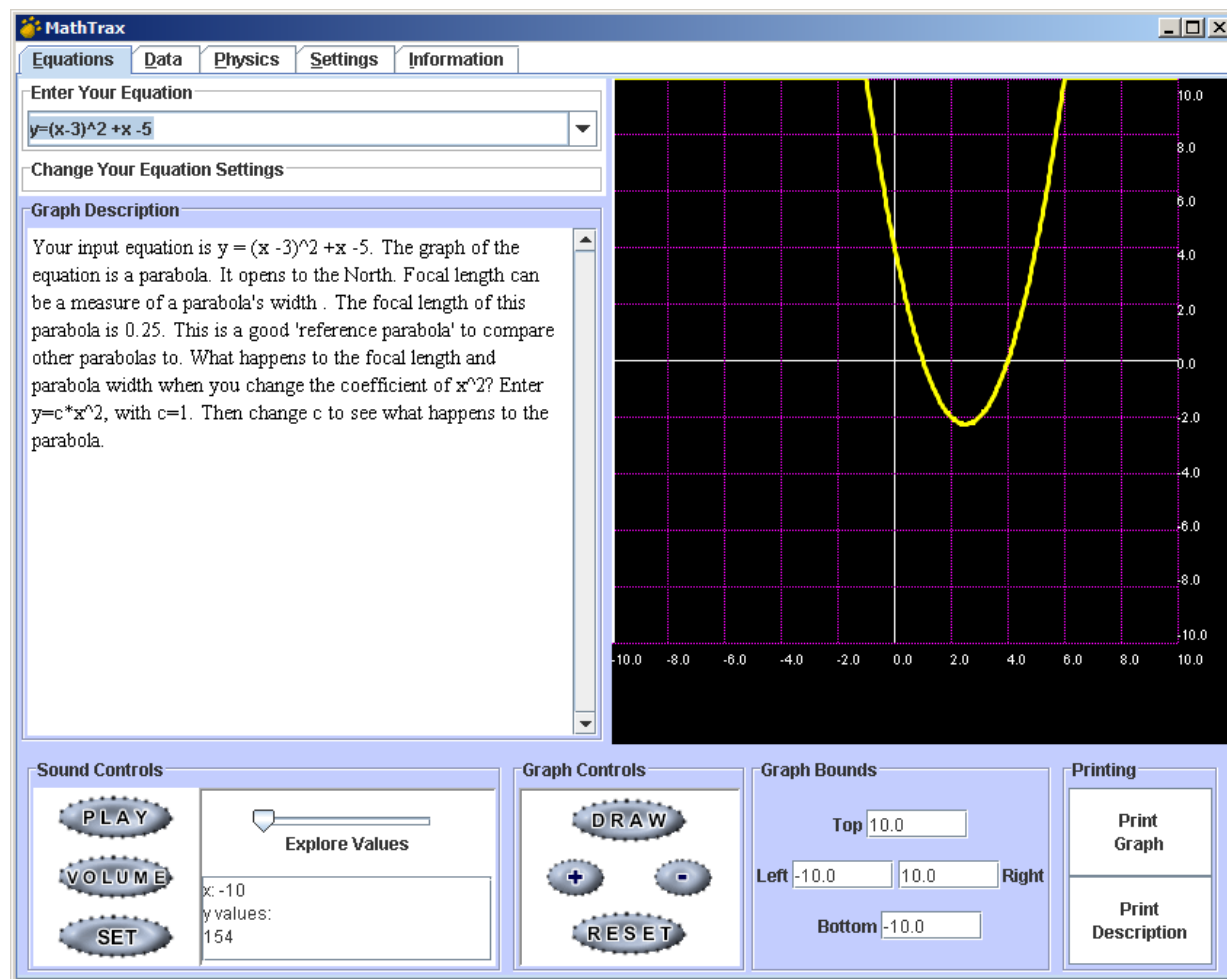
4. Framework of the Study

Research has shown that there are multiple reasons for educators to adopt various types of technology for use in the classroom for teaching and learning mathematics. Isiksal and Askar (2005) conducted a study on how the use of spreadsheets and dynamic geometry software affects the achievement of 7th grade students in Ankara, Turkey. The experiment had one group using Excel spread sheets, another using Autograph (a type of dynamic software), and the last group had the traditional style of teaching to learn different math concepts. The study found that the students who used Autograph had the highest scores out of the three groups regarding mathematics achievement. Hannafin and Scott (1998) conducted a study involving 210 eighth grade students who worked through 16 geometric activities using Geometer's Sketchpad. Through a test the researchers created that included lower order and higher order thinking questions, the researchers discovered that the students with lower high school math grades performed higher than students with higher math grades on the items involving conceptual understanding. Funkhouser (2002) conducted a study involving 49 high school geometry students that were split into a control group and an experimental group. The control group was taught plane geometry using the traditional style of lecturing while the experimental group was

taught plane geometry using the Geometric Supposer, a type of dynamic software. His study found that students who used Geometric Supposer had better performance on a standardized geometry test than the students who learned through traditional lecturing. Bos (2007) conducted research that first looked at the research of Willhelm, Confrey, Casro-Filho, and Maloney and discovered that when students use technology, they learn essential algebra concepts faster and in more depth. Bos also conducted her own study involving 95 eleventh grade students in Texas where ten classes received instruction on quadratic functions using the Texas Instrument InterActive instructional software and nine classes received traditional instruction. She used the Texas Assessment of Knowledge and Skills Grade 10 Mathematics Test scores to test the students' learning. She found that using this instructional software increased the students' mathematics achievement on quadratic functions (Bos, 2007). There have also been a few studies that have involved pre-service mathematics teachers. Pandiscio (2002) conducted a study involving 4 secondary pre-service mathematics teachers who used Geometer's Sketchpad to understand the concept of proof. Through interviews done by the researcher, he found that the students believed that using the dynamic software helped them understand more deeply the ideas from the theorems used than they would have not using the software. Guven and Kosa (2008) conducted a study of 40 pre-service mathematics teachers and had them use the dynamic geometry software Capri 3D to see if the technology affected their spatial skills. The students took a pre-test, used Capri 3D for 8 weeks, and then took the same test as a post test. The study found that the software intervention increased the students' spatial skills.

Importantly, a large amount of research confirms that "students' achievement is positively affected when they use curricula designed with graphing calculator as a primarily" (Kastbergs & Leatham, 2005, p. 25). From the profound study in 2006, Idris had claimed the use of graphing calculator injecting new excitement and enthusiasm into mathematics teaching and learning process. However, there arose multiple concerns about the use of hand-held calculator with the lack of students' understanding of basic mathematical concepts as well as the lack of interactive and collaborative learning (see Habre and Grudmeier (2007), and Monaghan (2004)). Upon those concerns, Cronk, Ferguson, and Patterson carried out a special research on tools for accessible math instruction, and projected that MathTrax graphical software does offer advantages over other accessible graphing calculator such as AGC (Accessible Graphing Calculator). MathTrax enables students to "interpret the form of the equation and automatically provides a text box describing some the chief characteristics of most forms of equations that middle and high school are likely to encounter (Cronk, Ferguson, and Patterson, 2006, see website). The screen capture on the next page illustrates how MathTrax plots and describes the characteristics of a quadratic function.

Adapting and extending from the above research, this study was carried out on the basis of experimenting and discovering the MathTrax tool in improving students' learning attitudes and promoting independent explorations through collaborative learning with peers. The following are the structures of the study.



5. Methodology

5.1. Participants

There were twenty eighth grade algebra students from a school in Northeast Ohio who participated in the study. Ten of them were randomly assigned to work individually on a computer. This group is called the MathTrax individual (MTI) group. The other ten worked with a partner also on the computer. This group is the MathTrax collaborative (MTC) group. The class was run in an exploratory set-up where students were free to ask questions to each other and to their teacher. In general, when one came into the class, one would notice the noise, the amount of activity, and the number of students involved in a debate or justifying their answer or method used to solve the problem. Students were given many mathematical challenges throughout the experiment and were encouraged to work together to solve problems as well as to get involved in the mathematical discussion. The activities were carried out in similar setting in that students were encouraged to interact together and naturally did so as they would during a normal class. For this study, there were fifteen computers in use for twenty students. All the students participated in this study were of a white Caucasian background with one Croatian who was bilingual. This random selected class was composed of seventeen females and three males.

All three males were assigned to the collaborative group and each worked with a female peer. Since the class was randomly selected based on the class time with the availability access of the computer lab for the teacher who participated in the STEM program to conduct the teaching with innovative technology sessions, the researcher did not collect information of how the student body of males and females were compiled in this class. It is worthwhile to add that STEM (Science, Technology, Engineering, Mathematics) program was initiated in 1992 with NASA and CAHSEE support to ensure that Hispanic Americans, female, and other under-represented minority students achieve academic excellence, and encourage them to pursue careers in engineering, science, and mathematics. Since the teacher of the class participated in the STEM program, it helped explain the reason why the class had a large amount of females.

5.2. Design of the study

The experiment was carried out in four class periods during the school year. MathTrax was used to allow the students to hands-on graphing basic functions that had already been taught in the lecture. This provided students with the first time opportunity of graphing basic algebraic functions using computer software. In other words, the study was designed to encourage the students to reinforce the graphing and function concepts they have already learned and at the same time to learn how to use a new computer dynamic software. Therefore, besides the rocket launching and roller coaster explorations, the examples used in this study were basic algebraic functions. During the first two periods, when the software was introduced, the students had opportunities to interact with the software, carry out experiments, and explore functions they already knew how to graph, and in general get acquainted to features in the software. During the last two periods, the students worked to complete activities which explored linear, quadratic functions, exponential functions, and transformations. Below are the sequences of activities of each class period along with the students' responses.

Students were told to meet in the regular classroom on the first period of the study rather than going straight to the computer lab. In the classroom, the teacher demonstrated to the students how the software interfaces and what the screen looks like, and talked about how to navigate through the different options. While students were told that there was sound feature to the software, the teacher did not play it or allude to specific sounds at this point. The students were asked to pick a linear graph they wanted to see, and they picked a function of their choice $y = 3x - 2$. First, the students were asked to graph it on their paper, and then the teacher graphed it with MathTrax. A comparison was made to confirm students' prediction. The teacher made a special point during this part of the exploration to highlight the graph descriptions part of MathTrax as students would need to use the information more during the second period's activities. The students were given a chance to make a slight adjustment to their function and predict what would change. The subsequent function they gave to the teacher was $y = -3x - 2$. One of the students stated that it would have a negative slope instead of a positive while another said the starting value (y -intercept) would remain the same. The students along with the teacher graphed it on the MathTrax and found the prediction to be correct. These simple examples were investigated primarily for the purpose of making students acquainted to the use of MathTrax software.

5.3 The activities

After the basic introduction to graphing with MathTrax, the students were blindfolded and asked to listen to the audio of their original function $y = 3x - 2$. After playing the sound for the graph of the function, the teacher asked the students to explain what they heard and what it meant. As observing, at first the students did struggle with identifying the separate sounds and distinguishing between what each meant. They were frustrated and just wanted to be able to look at the graph, but as the class went on with a series of simulated functions, they started to identify the different sounds. Switching between the first two functions $y = 3x - 2$ and $y = -3x - 2$, the students first picked up on the low to high pitch verses high to low pitch which indicated a positive verses negative slope. From this, they themselves drew a conclusion that higher pitches indicate higher y values and lower pitches indicate lower y values. The second aspect they picked up on was background noise which indicated the line was below the y -axis. They had a harder time with the fact that there was a *bell* sound every time the line crossed the y -axis. Many wanted to say the line crossed the y -axis when the background noise started or stopped, so the teacher decided to graph the function $y = -\frac{1}{2}x + 4$. For this graph, the x and y -intercept are more spread out; therefore, it is much easier to distinguish between the sounds, and most of them picked up on the sounds much faster. The students also struggled with the *chirp* which sounds when the line crosses the x -axis. Some of the students were still struggling even after listening to different linear functions. At this point, the teacher had the students take off the blind folds and watch the moving point on the graph as the sound played. This helped confirm their recognitions and the associations that had discussed with the whole class. The students were then asked to journal about the sounds and what they indicated.

The subsequent activity was a prediction and describing what sound should be produced from a given graph. It was indicated that the students found this easier to pick up than the previous blindfolded sound identification activity, although identifying when the x and y axis crossing sounds fit with the background noise did challenge them a bit until they sorted out when each happens within their own mind. After completing this activity as a whole group with the students predicting which sounds they should hear for a few different graphs, the class moved up to the computer lab. In the computer lab, all students were given a chance to play with graphs, zoom in and out, and play with the audio features. They were asked to focus on the timing and how quickly the sounds would start or stop. The third activity mandated them to use what they learned about the timing in a Guess the Function Game. This Function Game includes linear functions with transformations and various different forms of quadratics. For the MTC, one student in each pair was blind-folded and the other graphed five different functions and played the sound for each of these functions one at a time. The partner who was blind-folded was asked to guess each of the five functions. After guessing, the student graphing the functions would indicate what was correct or incorrect in each guess before playing the second function. Students then switched roles. The MTI students carried the same tasks but on individual study. This activity helped them apply everything they had learned throughout the two previous class periods with an emphasis on accuracy or attention to detail in the sounds they were hearing. It helped them to focus on the meaning of each sound rather than just playing the sounds because they are cool and fun and helped them put together everything they had discussed and explored.

On the third class period, the activity asked students to focus more on transformations to both the linear and quadratic graphs and spend less time just experimenting with the different features of MathTrax. The first task they had to complete was to determine how changing the slope or the y-intercept in a linear function (m or b in the form $y = mx + b$) affected the graph. They also looked at the effect of using opposite m values, reciprocal m values, and opposite reciprocal m values. Students were encouraged to use the sound feature throughout their exploration. Many students used the sound to help determine if lines were parallel because the speed of the change in tonal values had to be the same for two parallel lines. They also liked using the sound to identify perpendicular lines because the speeds were opposite (one slow, one fast, one increasing, and one decreasing). If both the speed and sound did not change, then the lines were not perpendicular. The researcher found that the students had been motivated in exploring this sound track themselves since they were neither prompted nor guided towards this discovery. The directions on the linear exploration were then discussed and the students were allowed to explore the effects of changing the slope or the y-intercept of linear equations both graphically and audibly.

In exploring the quadratic graphs, the students were given two different forms of quadratic functions, the standard form $y = ax^2 + bx + c$ and the vertex form $y = a(x - h)^2 + k$. The students knew very little of what to expect with these graphs. So, there was the normal confusion initially where students were unsure of where to start or what to do. However, once they started experimenting with different values, they started to see what each value controlled and how to get the graph to move in certain directions. They also started wondering if the graph always turned up or was a U-shape. The teacher prompted them by asking, "Have you tried with different integer values?" This pretty much reminded them to try the negative numbers although the students were reminded at times later in class to try negative numbers again. The class also discussed the importance of only changing one value at a time in order to determine how that specific change affects the graph. The students did well coming up with many conclusions which they could demonstrate but struggled to verbalize. Through discussing and showing these changes, the students worked through the process of describing their observations in mathematical terms. Towards the end of the discussion this verbalization became more natural. At the end of the discussion/activity, the students had to predict the shape, values, and sound for two different quadratic functions, using what they had learned and what had been discussed. This was their exit ticket and part of the evaluation for the study. The majority of students did well with this, although a couple struggled with the form $y = ax^2 + bx + c$. All of them did get that opposite a values opens the parabola in opposite direction. More time to explore and maybe more guided discovery would have helped them increase their accuracy in their descriptions of the graphs.

6. Influences of the use of MathTrax on student's learning

Overall, students' accomplishments were very similar across the groups. However, the MTC showed more focused on examining the challenging problems, such as rockets and roller coaster periodic functions. These students in pairs spent most of their time in exploring and challenging each other. Their discussion and questions were recorded as such: "Did you recognize the periodic rhythm of the slope?" "Hey, tell me when the slope is parallel to the x-axis," "What kind of line you plotted, why I only heard positive slopes?" or "Are you kidding? The angle of rocket shooting should be close to 90 degree."

The MTI students without partners were still discussing and sharing results with others and the teacher. But, they mainly spent their time working on the assigned problems rather than the explorations. Some of these students struggled with the open directions given which told them to explore what effect change the slope or the y -intercept or the coefficient values a , b , and c of the function $y = ax^2 + bx + c$, etc. had on the graph. To help this individual group, the teacher had to give a list of functions they could graph and describe the changes in each. This helped guide their exploration, still allowing them to make the discoveries rather than just telling them what they should observe. These students did significantly better with this guided discovery than open discovery as in the collaborative group.

At the end of the four class periods of the experiment, the students were asked to write their reflection on the use of the MathTrax software. Their statements were summarized and analyzed. Overall summary, students from both groups enjoyed using the software. It was the first time they were exposed to the computer technology which would graph for them. One student commented, "I could use this at home to check my homework." Another student joked about just using it to complete the homework, but recognized the need to do it by hand first in order to learn and understand what is happening in the graph and how each part of the function affects the graph. One male student fondly expressed that when he closed his eyes, discovering the graph function was the same as playing computer game with his peer. One female student especially liked the sound aspect of the program because it helped her think about the details of the graph in another way. She said, "I feel like I understand the graph better now that I've heard it. I picture an object falling into a hole." While all students from the MTC group enjoyed their explorations with partners, there exists a student from the MTI who were not so fond of MathTrax. On the first day of the experiment, all the students were generally more hesitant but starting by the second day, most students just took off with using the program. However, there was one MTI student never quite embraced it. This particular one commented, "If I get it and understand what the graph will look like why do I have to use MathTrax? I don't understand what the point of the sound it went. We can see it and you can tell what the graph will do from the equation anyway." This reaction was predictable because this student is a good student who often catches on quickly most of the function concepts and does not like to spend time doing things she already knows and has already figured out. She would rather do it by hand than deal with using the technology. During the experiment time, the teacher did allow her to use the technology to work with more complex ideas by having her explore the physics tab herself.

In summary, students from both groups enjoyed working with MathTrax. Their discussions, questions to either their peers or to the teacher, and their reflections by the end of the study revealed that the MTC students could get further with self explorations and showed better prospective in open discovery. The MTI needs the teacher's guidance to reach the discovery level.

7. Implications

After using MathTrax in Algebra I class for a couple of weeks, the teacher who participated in this study highly recommended that math teachers should look into using it in their classes, especially if they teach at a school where graphing calculators are not available and where the school does not have the funds to purchase graphing calculators. This program is free and has

excellent capabilities so similar and in some ways more powerful than a graphing calculator. It also works well for schools where teachers do not feel it is justifiable or reasonable to request that all the students purchase their own graphing calculator as MathTrax can be downloaded at home by each student and then used for homework assignments and projects which students are required to complete or work on both in class as well as outside of class time.

The result of this study conveyed that MathTrax is a valuable program. It is motivating and intriguing to students, it has multiple modes of exploring the graphs of many different types of functions which extend far beyond the algebra I curriculum that is usually taught. It has applications to rockets and roller coasters built in with free accessibility. This makes MathTrax software an extremely useful tool for schools to use which can be downloaded on school computers and on student computers at home to allow students to utilize the functionality of the program both in school and at home. Furthermore, MathTrax is a good tool for collaborative learning and independent discovery. The downside to this program is that it is not handy or as convenient as a graphing calculator which can sit on a student's desk in the typical classroom and be used throughout class a few times. The inconvenience of signing out the computer lab and holding the entire class period there restricts the use of MathTrax to more full class labs or exploration activities rather than allowing it to be a tool used throughout class. However, based on results of this study, it would be worth taking the time to use it for an entire class as student motivation and interest increased, student involvement increased, and students' attitudes improved. It is believed that a further study would need to be done with two similar groups of students side-by-side for a longer period of time. Additionally, students' explorations and discussions should be in written format for further analysis of collaborative and independent study ability. Based on class discussion and the student exit paths of this study, students are relating to the graphs much better since working with MathTrax. They actually see the graph as representing motion or actions rather than being just a curve line on the page which has limited meaning besides being a picture for a function.

8. References

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